motion in the positive direction, or if the vessel is extended during motion in the negative direction (membrane 4 is connected to the fixed wall by means of filament 5). The body is displaced in the negative direction if during its motion in the positive direction the vessel is extended, or if it is compressed during its motion in the negative direction (membrane 4 is connected to the fixed wall by means of filaments 6, 7 and batten 8).

Figure 2a, b shows two series of snapshots demonstrating the motion of the body along the Z axis with respect to the vessel when prescribed matched oscillations and deformation are imposed on the vessel. In Fig. 2a, the body is displaced in the positive direction; in Fig. 2b, it is displaced in the negative direction. The distance between adjacent vertical subdivisions is 0.5 cm (these scale divisions are fixed with respect to the vessel). The framing frequency is 5 shots/sec. The amplitude and period of the vessel oscillation is 0.3 cm and 0.25 sec, respectively.

Comparison of results given here and those of [1] shows that the predominantly unidirectional motion of the compressible solid body and a gas bubble are qualitatively the same. The predominantly unidirectional motion of the compressible solid body can be explained in the same way as the analogous motion of a gas bubble (see [1.2]). From this it may be concluded that there exists a phenomenon of predominantly unidirectional motion of a compressible inclusion in a vibrating liquid.

The author wishes to thank D. G. Akhmetov for discussions of issues related to this work.

LITERATURE CITED

1. V. L. Sennitskii, "Predominantly unidirectional motion of a gas bubble in a vibrating liquid," Dokl. Akad. Nauk SSSR, 319, No. 1 (1991).
2. V. L. Sennitskii, "On the motion of a gas bubble in a viscous vibrating liquid," Prikl. Mekh. Tekh. Fiz., No. 6 (1988).

ANALYTICAL SOLUTION OF THE ONE-DIMENSIONAL PROBLEM OF MODERATELY STRONG EVAPORATION (AND CONDENSATION) IN A HALF-SPACE

A. V. Latyshev and A. A. Yushkanov

UDC 533.72+517.958

We obtain for the first time an exact solution of the problem of evaporation (and condensation) of a liquid occupying a half-space and evaporating into a vacuum. We use the one-dimensional Boltzmann equation with the collision operator in the BGK (Bhatnagar—Gross—Krook) form, linearized about the equilibrium distribution function far from the surface between the phases.

The history of this problem, for which there is no available analytical solution even in the linear formulation using the one-dimensional BGK equation, has been described in [1, 2], where the exact solution of the problem was considered for so-called strong evaporation of a liquid into a vacuum. The problem was linearized about the equilibrium distribution function far from the surface of evaporation and the effect of the translational motion of the gas on its behavior in the Knudsen layer was taken into account in the linear approximation. The escape velocity of the gas (and other parameters) appear nonlinearly in the distribution function. This approach can be called quasilinear. In spite of its crudeness, it correctly describes a number of important qualitative features of evaporation such as the special position of the Mach number equal to unity.

In [1] the problem was solved using the resolvent method, in [2] it was reduced to a boundary-value problem, and in [3] the methods of functional analysis were used to show that the problem has a solution when $U < \sqrt{3/2}$ and does not have a solution when $U \geq \sqrt{3/2}$. Here

Moscow. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 1, pp. 102-108, January-February, 1993. Original article submitted June 27, 1991; revision submitted January 15, 1992.

U is the dimensionless velocity of the evaporating gas. The approximate F_N method was used in [4]. Finally, the problem was solved in an abstract formulation in [5, Chap. III, Sec. 4].

We consider evaporation (condensation) into a vacuum of a liquid with a plane surface x = 0. The liquid occupies the half-space x > 0. We take the one-dimensional BGK equation

$$\zeta \frac{\partial}{\partial x} f(x, \zeta) = \nu [\Phi(x, \zeta) - f(x, \zeta)], \tag{1}$$

where f(x, ζ) is the distribution function; ζ is the molecular velocity in the x direction; ν is the collision frequency; and Φ(x, ζ) is a local Maxwellian distribution

$$\Phi(x, \zeta) = \frac{\rho(x)}{\sqrt{2\pi RT(x)}} \exp\left\{-\frac{[\zeta - v(x)]^2}{2RT(x)}\right\}.$$

Here the density, mass velocity, and temperature are defined as

$$\rho(x) = \int_{-\infty}^{\infty} f(x, \zeta)\, d\zeta,\ \rho(x)\, v(x) = \int_{-\infty}^{\infty} \zeta f(x, \zeta)\, d\zeta.$$

$$\rho(x) RT(x) = \int_{-\infty}^{\infty} [\zeta - v(x)]^2 f(x, \zeta)\, d\zeta.$$

We assume that far from the surface the state of the vapor is described by the equilibrium distribution with constant velocity of evaporation (condensation) v_∞, density ρ_∞, and temperature T_∞:

$$\lim_{x\to\infty} \Phi(x, \zeta) = f_\infty(\zeta) = \frac{\rho_\infty}{\sqrt{2\pi RT_\infty}} \exp\left\{-\frac{(\zeta - v_\infty)^2}{2RT_\infty}\right\}.$$

Following [1], we linearize f(x, ζ) and Φ(x, ζ) in $f_\infty(\zeta)$. Introducing the variable $c = \zeta - v_\infty$, we write

$$f(x, c) = f_\infty(c)\ [1 + h(x, c)], \tag{2a}$$

where

$$f_\infty(c) = \frac{\rho_\infty}{\sqrt{2\pi RT_\infty}} \exp\left\{-\frac{c^2}{2RT_\infty}\right\}. \tag{2b}$$

Substituting (2a) into (1) and linearizing Φ(x, ζ) in $f_\infty(\zeta)$, we obtain the equation

$$(\mu + U)\frac{\partial}{\partial \bar{x}} h(\bar{x}, \mu) + h(\bar{x}, \mu) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu'^2} q(\mu, \mu')\, h(\bar{x}, \mu')\, d\mu' \tag{3}$$

with the boundary conditions

$$h(0,\ \mu) = -2U\mu + \varepsilon_n + \varepsilon_T(\mu^2 - 1/2),\ \ \mu > -U,\ \ h(\infty,\ \mu) = 0. \tag{4}$$

Here $q(\mu, \mu') = 1 + 2\mu\mu' + 2(\mu^2 - 1/2)(\mu'^2 - 1/2)$ is the kernel of the equation; $\bar{x} = \nu x(2RT_\infty)^{-1/2}$; $\mu = c(2RT_\infty)^{-1/2}$; $U = v_\infty(2RT_\infty)^{-1/2}$; ε_T and ε_n are the temperature and density jumps; U is the velocity of evaporation (U > 0) or condensation (U < 0). The variable $\bar{x}$ will be replaced by x.

The Case ansatz [6] $h(x, \mu) = \varphi(\eta, \mu)\exp(-x/(\eta + U))$ immediately reduces (3) to the characteristic equation

$$(\eta - \mu)\varphi(\eta, \mu) = (\eta + U)\frac{1}{\sqrt{\pi}}\Big\{n^{(0)}(\eta) + 2\mu n^{(1)}(\eta) + \\ + 2\left(\mu^2 - \frac{1}{2}\right)\left[n^{(2)}(\eta) - \frac{1}{2} n^{(0)}(\eta)\right]\Big\}, \tag{5}$$

where

$$n^{(k)}(\eta)=\int_{-\infty}^{\infty} e^{-\mu^2}\varphi(\eta,\mu)\,\mu^k d\mu \quad (k=0,1,2).$$

Multiplying (5) by $\mu^k \exp(-\mu^2)$ ($k = 0.1$) and integrating with respect to μ in $\mathbf{R} = (-\infty, +\infty)$, we obtain

$$n^{(1)}(\eta) = -Un^{(0)}(\eta) \text{ and } n^{(2)}(\eta) = -Un^{(1)}(\eta)\,.$$

Then (5) can be written in the form

$$(\eta-\mu)\varphi(\eta,\mu) = \pi^{-1/2}(\eta+U)q(-U,\mu)n(\eta). \tag{6}$$

Here $q(-U,\mu) = 1 - 2U\mu + 2(U^2 - 1/2)(\mu^2 - 1/2)$; $n(\eta) \equiv n^{(0)}(\eta)$. The characteristic equation (6) has eigenfunctions of the continuous spectrum

$$\varphi(\eta,\mu)=\pi^{-1/2}(\eta+U)\,q(-U,\mu)\,P\frac{1}{\eta-\mu}\,n(\eta)+g(\eta)\,\delta(\eta-\mu), \tag{7}$$

where $g(\eta)$ is determined from the normalization condition

$$n(\eta)=\int_{-\infty}^{\infty} e^{-\mu^2}\varphi(\eta,\mu)\,d\mu;$$

the symbol $P\frac{1}{x}$ denotes the principal value of the integral of the function $1/x$; $\delta(x)$ is the Dirac delta function; η and $\mu \in \mathbf{R}$.

Substituting (7) into the above equation we have

$$g(\eta)=e^{\eta^2}\lambda(\eta)\,n(\eta). \tag{8}$$

Here

$$\lambda(z)=1+\pi^{-1/2}(z+U)\int_{-\infty}^{\infty}\exp(-\mu^2)\frac{q(-U,\mu)}{\mu-z}\,d\mu$$

is the dispersion function.

It can be shown [7] that the dispersion function $\lambda(z)$ does not have finite complex zeros. We expand it in a Laurent series about $z = \infty$:

$$\lambda(z)=-U\left(U^2-\frac{3}{2}\right)z^{-3}-\frac{3}{2}\left(U^2-\frac{1}{2}\right)z^{-4}-3U\left(U^2-\frac{3}{2}\right)z^{-5}+\dots \tag{9}$$

We see from the expansion (9) that the point $z = \infty$ is a third-order zero of the dispersion function if $U \neq 0$ and $U^2 \neq 3/2$ and it is a fourth-order zero if $U = 0$ or $U^2 = 3/2$. This point (as a multiple point of the discrete spectrum) corresponds to the following discrete modes:

$$\begin{aligned} h_\alpha(x,\mu) &= \mu^\alpha,\ \alpha=0,1,2 \ (\text{if} \quad U\neq 0 \text{ and } U^2\neq 3/2),\\ h_3(x,\mu) &= (x-U-\mu)q(-U,\mu) \ (\text{if} \quad U=0 \ \text{ or } \ U^2=3/2). \end{aligned} \tag{10}$$

Knowing the eigenfunctions of the continuous (7) and discrete (10) spectra, the general solution of (3) is written as an integral over the continuous spectrum and a sum of discrete eigenfunctions:

$$h(x,\mu)=\sum_{\alpha=0}^{\varkappa}A_\alpha h_\alpha(x,\mu)+\int_{-\infty}^{\infty}\varphi(\eta,\mu)\exp[-x/(\eta+U)]\,d\eta. \tag{11}$$

Here $\kappa = 2$ for $U^2 \neq 0, 3/2$ and $\kappa = 3$ for $U^2 = 0, 3/2$. The constants A_α ($\alpha = 0, 1, \ldots, \kappa$) and the function $n(\eta)$ are the expansion coefficients of the solution and are determined from the boundary conditions imposed on $h(x, \mu)$.

Taking into account the boundary conditions (4), the expansion (11) is reduced to the integral equation

$$h(0, \mu) = \int_{-U}^{\infty} \varphi(\eta, \mu)\, d\eta, \quad \mu > -U. \tag{12}$$

Substituting (7) into (12), we obtain a singular integral equation with Cauchy kernel

$$h(0, \mu) = e^{\mu^2}\lambda(\mu)\, n(\mu) + \pi^{-1/2} q(-U, \mu) \int_{-U}^{\infty} \frac{(\eta + U)\, n(\eta)}{\eta - \mu}\, d\eta. \tag{13}$$

We note that

$$\lambda^+(\mu) - \lambda^-(\mu) = 2\pi^{1/2}(\mu + U)\, e^{-\mu^2} q(-U, \mu),$$

and introduce the function

$$N(z) = \frac{1}{2\pi i} \int_{-U}^{\infty} \frac{(\eta + U)\, n(\eta)\, d\eta}{\eta - z}, \tag{14}$$

for which we have on the cut $\mathbf{R}_U = (-U, +\infty)$

$$N^+(\mu) - N^-(\mu) = (\mu + U) n(\mu),$$

$$N^+(\mu) + N^-(\mu) = \frac{1}{\pi i} \int_{-U}^{\infty} \frac{(\eta + U)\, n(\eta)}{\eta - \mu}\, d\eta.$$

Multiplying both sides of (13) by $(\mu + U)e^{-\mu^2}$, this equation reduces to the Riemann boundary-value problem [7]

$$\lambda^+(\mu)N^+(\mu) - \lambda^-(\mu)N^-(\mu) = (\mu + U)\exp(-\mu^2)h(0, \mu), \quad \mu > -U. \tag{15}$$

Multiplying both sides of (15) by $2\pi^{1/2} i q(-U, \mu)$ and using the boundary values of $\lambda(z)$ on the real axis from above and below, (15) is transformed to

$$\begin{aligned} &\lambda^+(\mu)\,[2\pi^{1/2} i q(-U, \mu) N^+(\mu) - h(0, \mu)] = \\ &= \lambda^-(\mu)\,[2\pi^{1/2} i q(-U, \mu) N^-(\mu) - h(0, \mu)], \quad \mu > -U. \end{aligned} \tag{16}$$

We consider the factorization of the coefficient of the boundary condition (16) [7]:

$$\frac{X^+(\mu)}{X^-(\mu)} = \frac{\lambda^+(\mu)}{\lambda^-(\mu)}, \quad \mu > -U. \tag{17}$$

Using (17), the boundary condition (16) is transformed to

$$\begin{aligned} &X^+(\mu)\,[2\pi^{1/2} i q(-U, \mu) N^+(\mu) - h(0, \mu)] = \\ &= X^-(\mu)\,[2\pi^{1/2} i q(-U, \mu) N^-(\mu) - h(0, \mu)], \quad \mu > -U. \end{aligned} \tag{18}$$

The solution of (18) depends on the solution of (17) in an essential way. We consider the solution of (17) as a function of the parameter U. We note that

$$q(-U, -U) = 2\left(U^2 - \frac{1}{2}\right)^2 + 2U^2 + 1 > 0, \quad q(-U, 0) = -\left(U^2 - \frac{3}{2}\right).$$

The function $q(-U, \mu)$ has two real roots

$$\mu_{1,2} = \frac{U \pm \sqrt{D(U)}}{2\left(U^2 - \frac{1}{2}\right)}.$$

Here $D(U) = 2[(U^2 - 3/4)^2 + 3/16] > 0$. We also have

$$\lambda^+(\mu) = \lambda(\mu) + \pi^{1/2} i(\mu + U) q(-U, \mu) \exp(-\mu^2).$$

1. Suppose $U \geq \sqrt{3/2}$. Then $\lambda(\mu) < 0$ when $\mu \to \infty$. It is not difficult to show that $[\theta(\mu)]_{R_U} = 3\pi$, where $\theta(\mu) = \arg \lambda^+(\mu)$, and $[\theta(\mu)]_{R_U}$ denotes the change in the function $\theta(\mu)$ when μ goes from $-U$ to $+\infty$. The solution of (17) which is bounded at $z = -U$ is given by [7]

$$X(z) = (z + U)^{-3} \exp\left\{\frac{1}{\pi} \int_{-U}^{\infty} [\theta(\tau) - 3\pi] \frac{d\tau}{\tau - z}\right\}.$$

It is now evident that (18) has only the trivial solution

$$2\pi^{1/2} i N(z) = h(0, z)/q(-U, z),$$

which cannot be used for the function $N(z)$ defined by (14), since this function is finite at infinity, whereas (14) vanishes at infinity.

2. Suppose $0 \leq U \leq \sqrt{3/2}$. Here $\lambda(\mu) > 0$ when $\mu \to \infty$. It can be shown that $[\theta(\mu)]_{R_U} = 2\pi$. The solution of (17) which is bounded at $z = -U$ is given by [7]

$$X(z) = (z + U)^{-2} \exp\left\{\frac{1}{\pi} \int_{-U}^{\infty} [\theta(\tau) - 2\pi] \frac{d\tau}{\tau - z}\right\}.$$

The general solution of (18) is now

$$2\pi^{1/2} i N(z) = (h(0, z) + c_0/X(z))/q(-U, z) \tag{19}$$

(c_0 is an arbitrary constant). The solution (19) is a meromorphic function, since $q(-U, z)$ has two real zeros μ_1 and μ_2. The poles of $N(z)$ at these points are eliminated by the conditions

$$h(0, \mu_\alpha) + c_0/X(\mu_\alpha) = 0, \quad \alpha = 1,2. \tag{20}$$

The function $N(z)$ vanishes at infinity if

$$c_0 = -\varepsilon_T, \tag{21}$$

which follows by expanding the right hand side of (13) in a Laurent series in negative powers of z.

Because $N(z)$ is defined in the complex plane, its limiting values $N^{\pm}(\mu)$ from above and below also have simple poles at the points μ_1 and μ_2. They are eliminated by imposing the four equations

$$h(0, \mu_\alpha) + c_0/X^{\pm}(\mu_\alpha) = 0, \quad \alpha = 1, 2. \tag{22}$$

We show that these conditions reduce to (20), i.e., they are satisfied automatically. We present without derivation the following integral representation for $X(z)$:

$$X(z) = \frac{1}{\sqrt{\pi}} \int_{-U}^{\infty} e^{-\mu^2} \gamma(\mu) \frac{q(-U, \mu)}{\mu - z} d\mu, \tag{23}$$

where $\gamma(\mu) = (\mu + U) X^+(\mu)/\lambda^+(\mu)$. It is evident from (23) that because the density of this integral vanishes at μ_1 and μ_2, the limiting values of the integral from above and below are equal to the values of the singular integral itself at these points. Hence $X^{\pm}(\mu_\alpha) = X(\mu_\alpha)$, $\alpha = 1, 2$, and (22) is satisfied automatically.

The temperature and density jumps can be found from (20) and (21) as functions of the velocity of evaporation U:

$$\varepsilon_T = 2U\frac{(\mu_1-\mu_2)X(\mu_1)X(\mu_2)}{(\mu_1^2-\mu_2^2)X(\mu_1)X(\mu_2)+X(\mu_1)-X(\mu_2)},$$
$$\varepsilon_\rho = 2U\mu_1-\left(\mu_1^2-1/2-1/X(\mu_1)\right)\varepsilon_T.$$

We consider the different cases of condensation.

1. Suppose $-\sqrt{3/2} < U < 0$. Then $\lambda(\mu) < 0$ when $\mu \to \infty$ and $[\theta(\mu)]_{R_U} = \pi$. Therefore the solution of (17) which is bounded at $z = -U$ can be taken as

$$X(z) = (z+U)^{-1}\exp\left\{\frac{1}{\pi}\int_{-U}^{\infty}[\theta(\tau)-\pi]\frac{d\tau}{\tau-z}\right\}.$$

Now the general solution of (18) is

$$2\pi^{1/2}iN(z) = (h(0,\ z)+(c_0+c_1z)/X(z))/q(-U,\ z).$$

From the condition that this solution must vanish at infinity we have $c_1 = -\varepsilon_T$ and to eliminate the poles at μ_1 and μ_2 we must have $X(\mu_\alpha)h(0,\ \mu_\alpha) + c_0 + c_1\mu_\alpha = 0$, $\alpha = 1, 2$. Hence

$$c_0 = \frac{1}{2}\left\{\varepsilon_T\left[\mu_1+\mu_2-X(\mu_1)\left(\mu_1^2-\frac{1}{2}\right)-X(\mu_2)\left(\mu_2^2-\frac{1}{2}\right)\right]+\right.$$
$$\left.+\,2U\,[\mu_1X(\mu_1)+\mu_2X(\mu_2)]-\varepsilon_n(X(\mu_1)+X(\mu_2))\right\},$$
$$\varepsilon_T = 2Uf(\mu_1,\ \mu_2,\ U)-\varepsilon_n g(\mu_1,\ \mu_2,\ U).$$

Here

$$f = [\mu_1X(\mu_1)-\mu_2X(\mu_2)]/\varphi(\mu_1,\ \mu_2,\ U);$$
$$g = (X(\mu_1)-X(\mu_2))/\varphi(\mu_1,\ \mu_2,\ U);$$
$$\varphi = \mu_2-\mu_1+X(\mu_1)(\mu_1^2-1/2)-X(\mu_2)(\mu_2^2-1/2).$$

2. Suppose $U < -\sqrt{3/2}$. Then $q(-U,\ \mu) > 0$ for all $\mu \geq -U$ and $\lambda(\mu) > 0$ when $\mu \to \infty$. It is not difficult to show that $[\theta(\tau)]_{R_U} = 0$. Therefore the solution of (17) which is bounded at $z = -U$ is

$$X(z) = \exp\left\{\frac{1}{\pi}\int_{-U}^{\infty}\theta(\tau)\frac{d\tau}{\tau-z}\right\}.$$

The general solution of (18) is then

$$2\pi^{1/2}iN(z) = [h(0,\ z)+(c_0+c_1z+c_2z^2)/X(z)]/q(-U,\ z)$$

(c_0, c_1, c_2 are arbitrary constants). From the condition that the solution must vanish at infinity we find $c_2 = -\varepsilon_T$ and to eliminate the poles we must have

$$X(\mu_\alpha)h(0,\ \mu_\alpha)+c_0+c_1\mu_\alpha+c_2\mu_\alpha^2 = 0,\quad \alpha = 1,\ 2,$$

from hence

$$c_1 = \varepsilon_T(\mu_1+\mu_2)-[X(\mu_1)h(0,\ \mu_1)-X(\mu_2)h(0,\ \mu_2)]/(\mu_1-\mu_2),$$
$$c_0 = \frac{1}{2}\{-c_1(\mu_1+\mu_2)+\varepsilon_T(\mu_1^2+\mu_2^2)-X(\mu_1)h(0,\mu_1)-X(\mu_2)h(0,\mu_2)\}.$$

It follows from these last two equations that the three parameters U, ε_T, and ε_ρ are necessary to uniquely specify the solution to the condensation problem.

Note. Our analysis shows that the evaporation and condensation problems are not symmetric from both the physical and mathematical points of view. In the problem considered here a Mach number of unity corresponds to a velocity of evaporation (condensation) $U = \sqrt{3/2}$. It follows from our results that this value plays a crucial role in evaporation and condensation. The results of the numerical calculations of [8] correspond to steady-state condensation for different U. A special case of the problem treated here was considered in [9] (the condensation problem). After solving (17) for the factorization of the coefficient, the authors of [9] were not able to carry through the solution to completion.

Finally we briefly describe the mathematical aspects of the method described here. The physical problem corresponds to solving a boundary-value problem for a simplified Boltzmann equation with a collision operator in the BGK form. The required physical quantities are contained in the boundary conditions. Separation of variables using the Case method leads to a characteristic equation whose eigenfunctions are generalized functions. Next the existence and uniqueness of the expansion of the solution of the boundary-value problem in continuous and discrete eigenfunctions is proved. The proof reduces to the solution of a singular integral equation with a Cauchy kernel, and this is reduced to the solution of a Riemann boundary-value problem on a half axis. After factorization of the coefficient of the boundary-value problem the general solution of the problem is found. The solution depends on the velocity of evaporation (condensation) in an essential way. The required physical quantities are determined from the solvability conditions for the boundary-value problem.

LITERATURE CITED

1. M. D. Arthur and C. Cercignani, "Nonexistence of a steady rarefied supersonic flow in a half-space," J. Appl. Math. Phys., 31, No. 5 (1980).
2. C. E. Siewert and J. R. Thomas, "Strong evaporation into a half-space," J. Appl. Math. Phys., 32, No. 4 (1981).
3. S. K. Loyalka, C. E. Siewert, and J. R. Thomas, "An approximate solution concerning strong evaporation into a half-space," J. Appl. Math. Phys., 32, No. 6 (1981).
4. W. Greenberg and C. V. M. van der Mee, "An abstract approach to evaporation models in rarefied gas dynamics," J. Appl. Math. Phys., 35, No. 2 (1984).
5. W. Greenberg, C. V. M. van der Mee, and V. Protopopescu, Boundary-Value Problems in Abstract Kinetic Theory, Birkhauser Verlag, Basel (1987).
6. M. Case and F. Zweifel, Linear Transport Theory, Addison-Wesley, Reading, Mass. (1967).
7. F. D. Gakhov, Boundary-Value Problems [in Russian], Nauka, Moscow (1977).
8. A. A. Abramov and M. N. Kogan, "Supersonic condensation of a gas," Dokl. Akad. Nauk SSSR, 278, No. 6 (1984).
9. C. Cercignani and A. Frezzotti, "Linearized analysis of a one-speed BGK model in the case of strong condensation," in: Theoretical and Applied Mechanics, Bulgarian Academy of Sciences, Vol. XIX, No. 3, Sofia (1988).